



WORKSHEET

Ungrouped Mean and Variance Question

Question Description: In this question you will commonly see expressions of the form $\sum(x - k)$ and $\sum(x - k)^2$. You will then have to values of the mean, standard deviation or variance. Below are the syllabus equations for the mean and standard deviation of ungrouped data.

(Note in the Representing data question worksheets you had plenty of examples of finding the mean and standard deviation of grouped data)

Syllabus

For ungrouped data:

$$\bar{x} = \frac{\sum x}{n}, \quad \text{standard deviation} = \sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Coding for Mean and Variance

$$\text{Mean}(x - c) = \text{Mean}(x) - c$$

$$\overline{(x - c)} = \bar{x} - c$$

$$\text{Var}(x - c) = \text{Var}(x)$$

$$\sigma^2(x - c) = \sigma^2(x)$$

Combined Mean and Variance

For data sets x and y with n_x and n_y respectively:

$$\text{Combined Mean} = \frac{\sum x + \sum y}{n_x + n_y}$$

$$\text{Combined Variance} = \frac{\sum x^2 + \sum y^2}{n_x + n_y} - \left(\frac{\sum x + \sum y}{n_x + n_y}\right)^2$$

QUESTIONS

Q1 w2015_p62

For n values of the variable x , it is given that $\Sigma(x - 100) = 216$ and $\Sigma x = 2416$. Find the value of n . [3]

Q2 w2015_p63

The time taken, t hours, to deliver letters on a particular route each day is measured on 250 working days. The mean time taken is 2.8 hours. Given that $\Sigma(t - 2.5)^2 = 96.1$, find the standard deviation of the times taken. [3]

Q3 w2017_p62

Andy counts the number of emails, x , he receives each day and notes that, over a period of n days, $\Sigma(x - 10) = 27$ and the mean number of emails is 11.5. Find the value of n . [3]

Q4 w2017_p63

Tien measured the arm lengths, x cm, of 20 people in his class. He found that $\Sigma x = 1218$ and the standard deviation of x was 4.2. Calculate $\Sigma(x - 45)$ and $\Sigma(x - 45)^2$. [3]

Q5 s2018_p61

In a statistics lesson 12 people were asked to think of a number, x , between 1 and 20 inclusive. From the results Tom found that $\Sigma x = 186$ and that the standard deviation of x is 4.5. Assuming that Tom's calculations are correct, find the values of $\Sigma(x - 10)$ and $\Sigma(x - 10)^2$. [3]

Q6 w2015_p61

- (a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

Q7 s2016_p63

The monthly rental prices, \$ x , for 9 apartments in a certain city are listed and are summarised as follows.

$$\Sigma(x - c) = 1845 \qquad \Sigma(x - c)^2 = 477\,450$$

The mean monthly rental price is \$2205.

- (i) Find the value of the constant c . [2]
- (ii) Find the variance of these values of x . [2]
- (iii) Another apartment is added to the list. The mean monthly rental price is now \$2120.50. Find the rental price of this additional apartment. [2]

Q8 s2017_p61

Kadijat noted the weights, x grams, of 30 chocolate buns. Her results are summarised by

$$\Sigma(x - k) = 315, \qquad \Sigma(x - k)^2 = 4022,$$

where k is a constant. The mean weight of the buns is 50.5 grams.

- (i) Find the value of k . [2]
- (ii) Find the standard deviation of x . [2]

Q9 w2017_p61

The ages of a group of 12 people at an Art class have mean 48.7 years and standard deviation 7.65 years. The ages of a group of 7 people at another Art class have mean 38.1 years and standard deviation 4.2 years.

- (i) Find the mean age of all 19 people. [2]
- (ii) The individual ages in years of people in the first Art class are denoted by x and those in the second Art class by y . By first finding Σx^2 and Σy^2 , find the standard deviation of the ages of all 19 people. [4]

Q10 w2018_p62

The Quivers Archery club has 12 Junior members and 20 Senior members. For the Junior members, the mean age is 15.5 years and the standard deviation of the ages is 1.2 years. The ages of the Senior members are summarised by $\Sigma y = 910$ and $\Sigma y^2 = 42\,850$, where y is the age of a Senior member in years.

- (i) Find the mean age of all 32 members of the club. [2]
- (ii) Find the standard deviation of the ages of all 32 members of the club. [4]

Q11 w2019_p61

The mean and standard deviation of 20 values of x are 60 and 4 respectively.

- (i) Find the values of Σx and Σx^2 . [3]

Another 10 values of x are such that their sum is 550 and the sum of their squares is 40 500.

- (ii) Find the mean and standard deviation of all these 30 values of x . [4]

Q12 s2015_p61

The table shows the mean and standard deviation of the weights of some turkeys and geese.

	Number of birds	Mean (kg)	Standard deviation (kg)
Turkeys	9	7.1	1.45
Geese	18	5.2	0.96

- (i) Find the mean weight of the 27 birds. [2]
- (ii) The weights of individual turkeys are denoted by x_t kg and the weights of individual geese by x_g kg. By first finding Σx_t^2 and Σx_g^2 , find the standard deviation of the weights of all 27 birds. [5]

Q13 s2018_p63

Farfield Travel and Lacket Travel are two travel companies which arrange tours abroad. The numbers of holidays arranged in a certain week are recorded in the table below, together with the means and standard deviations of the prices.

	Number of holidays	Mean price (\$)	Standard deviation (\$)
Farfield Travel	30	1500	230
Lacket Travel	21	2400	160

- (i) Calculate the mean price of all 51 holidays. [2]
- (ii) The prices of individual holidays with Farfield Travel are denoted by $\$x_F$ and the prices of individual holidays with Lacket Travel are denoted by $\$x_L$. By first finding Σx_F^2 and Σx_L^2 , find the standard deviation of the prices of all 51 holidays. [5]

Q14 s2019_p61

The times, t seconds, taken to swim 100 m were recorded for a group of 9 swimmers and were found to be as follows.

95 126 117 135 120 125 114 119 136

- (i) Find the values of $\Sigma(t - 120)$ and $\Sigma(t - 120)^2$. [2]
- (ii) Using your values found in part (i), calculate the variance of t . [2]

Q15 w2018_p63

The heights, in cm, of the 11 members of the Anvils athletics team and the 11 members of the Brecons swimming team are shown below.

Anvils	173	158	180	196	175	165	170	169	181	184	172
Brecons	166	170	171	172	172	178	181	182	183	183	192

The heights of the 11 members of the Anvils are denoted by x cm. It is given that $\Sigma x = 1923$ and $\Sigma x^2 = 337\,221$. The Anvils are joined by 3 new members whose heights are 166 cm, 172 cm and 182 cm.

(iii) Find the standard deviation of the heights of all 14 members of the Anvils. [4]

ANSWERS

$$\textcircled{1} \quad \sum (x - 100) = 216$$

$$\sum x = 2416$$

$$\bar{x} = \frac{\sum x}{n} = \frac{2416}{n}$$

$$\therefore \boxed{\bar{x} = \frac{2416}{n}} \quad \text{Eq 1}$$

$$\overline{(x - 100)} = \frac{\sum (x - 100)}{n} = \frac{216}{n}$$

$$\therefore \overline{(x - 100)} = \frac{216}{n}$$

As we know : $\overline{(x - 100)} = \bar{x} - 100$

$$\therefore \boxed{\bar{x} - 100 = \frac{216}{n}} \quad \text{Eq 2}$$

$$\text{Eq 1} \rightarrow \text{Eq 2} : \left(\frac{2416}{n} \right) - 100 = \frac{216}{n}$$

$$2416 - 100n = 216$$

$$100n = 2200$$

$$\boxed{n = 22}$$

$$\textcircled{2} \quad n = 250$$

$$\bar{t} = 2.8$$

$$\sum (t - 2.5)^2 = 96.1$$

$$\sigma^2(x) = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\text{As } \sigma^2(t - 2.5) = \sigma^2(t)$$

$$\therefore \frac{\sum (t - 2.5)^2}{n} - (\bar{t} - 2.5)^2 = \sigma^2(t)$$

$$\frac{96.1}{250} - (\bar{t} - 2.5)^2 = \sigma^2(t)$$

$$\therefore \frac{96.1}{250} - (2.8 - 2.5)^2 = \sigma^2(t)$$

$$\therefore \sigma^2(t) = 0.2944$$

$$\therefore \boxed{\sigma(t) = 0.543}$$

$$\textcircled{3} \quad \sum (x-10) = 27 \quad \bar{x} = 11.5$$

$$\overline{(x-10)} = \frac{\sum (x-10)}{n}$$

$$\text{As } \overline{(x-10)} = \bar{x} - 10 = 11.5 - 10 = 1.5$$

$$\therefore 1.5 = \frac{27}{n}$$

$$\boxed{n = 18}$$

$$\textcircled{4} \quad n = 20 \quad \sum (x-45) = ?$$

$$\sum x = 1218$$

$$\sum (x-45)^2 = ?$$

$$\sigma(x) = 4.2$$

$$\overline{(x-45)} = \frac{\sum (x-45)}{n}$$

$$\text{As } \overline{(x-45)} = \bar{x} - 45$$

$$\therefore \bar{x} - 45 = \frac{\sum (x-45)}{n}$$

$$\left(\frac{1218}{20} \right) - 45 = \frac{\sum (x-45)}{20}$$

$$\therefore \boxed{\sum (x-45) = 318}$$

$$\sigma(x-45) = \sigma(x)$$

$$\sqrt{\frac{\sum (x-45)^2}{20} - \left(\frac{\sum (x-45)}{20}\right)^2} = 4.2$$

$$\frac{\sum (x-45)^2}{20} - \left(\frac{318}{20}\right)^2 = (4.2)^2$$

$$\therefore \boxed{\sum (x-45)^2 = 5409}$$

$$\textcircled{5} \quad n=12 \quad \sum x = 186 \quad \sigma(x) = 4.5$$

$$\overline{(x-10)} = \bar{x} - 10$$

$$\frac{\sum (x-10)}{n} = \left(\frac{\sum x}{n} \right) - 10$$

$$\begin{aligned} \sum (x-10) &= \sum x - 10n \\ &= 186 - 10(12) \end{aligned}$$

$$\therefore \boxed{\sum (x-10) = 66}$$

$$\sigma(x-10) = \sigma(x)$$

$$\sqrt{\frac{\sum (x-10)^2}{n} - \overline{(x-10)}^2} = \sigma(x)$$

$$\frac{\sum (x-10)^2}{12} - \left(\frac{66}{12} \right)^2 = (4.5)^2$$

$$\therefore \boxed{\sum (x-10)^2 = 606}$$

$$\textcircled{6} \quad \sum (x-80) = -147 \quad \sum (x-80)^2 = 952$$

$$n = 30$$

$$\overline{(x-80)} = \bar{x} - 80$$

$$\therefore \frac{\sum (x-80)}{n} = \bar{x} - 80$$

$$\frac{-147}{30} + 80 = \bar{x}$$

$$\boxed{\bar{x} = 75.1}$$

$$\sigma(x-80) = \sigma(x)$$

$$\therefore \sqrt{\frac{\sum (x-80)^2}{n} - (\overline{(x-80)})^2} = \sigma(x)$$

$$\therefore \sigma(x) = \sqrt{\frac{952}{30} - \left(\frac{-147}{30}\right)^2}$$

$$\boxed{\sigma(x) = 2.78}$$

$$\textcircled{7} \quad n=9 \quad \sum (x-c) = 1845$$

$$\sum (x-c)^2 = 477,450$$

$$\bar{x} = 2205$$

(i)

$$\overline{(x-c)} = \bar{x} - c$$

$$\frac{\sum (x-c)}{n} = \bar{x} - c$$

$$c = \bar{x} - \frac{\sum (x-c)}{n}$$

$$= 2205 - \frac{1845}{9}$$

$$\boxed{c = 2000}$$

(ii)

$$\sigma^2(x-c) = \sigma^2(x)$$

$$\therefore \sigma^2(x) = \frac{\sum (x-c)^2}{n} - \overline{(x-c)}^2$$

$$= \frac{477,450}{9} - \left(\frac{1845}{9} \right)^2$$

$$= 11,025$$

$$\boxed{\sigma^2(x) = 11,025}$$

$$(iii) \quad \bar{x}_{old} = 2205 \quad \bar{x}_{new} = 2120.5$$
$$\bar{x}_{new} = \frac{\sum x_n}{n_1} \quad n_1 = 10$$
$$n_2 = 9$$

$$\sum x_n = (10)(2120.5)$$
$$= 21,205$$

$$\bar{x}_{old} = \frac{\sum x}{n_2}$$

$$\sum x = (9)(2205)$$
$$= 19845$$

$$\therefore \text{New Apartment} = \sum x_n - \sum x$$
$$= 21,205 - 19845$$
$$= 1360$$

$$\boxed{\text{New Apartment} = \$1360}$$

$$\textcircled{8} \quad n = 30 \quad \sum (x - k) = 315$$

$$\bar{x} = 50.5 \quad \sum (x - k)^2 = 4022$$

$$(i) \quad \overline{(x - k)} = \bar{x} - k$$

$$\frac{\sum (x - k)}{n} = \bar{x} - k$$

$$\frac{315}{30} = 50.5 - k$$

$$\boxed{k = 40}$$

$$(ii) \quad \sigma(x - k) = \sigma(x)$$

$$\therefore \sigma(x) = \sqrt{\frac{\sum (x - k)^2}{n} - (\overline{(x - k)})^2}$$

$$= \sqrt{\frac{4022}{30} - \left(\frac{315}{30}\right)^2}$$

$$\therefore \boxed{\sigma(x) = 4.88}$$

$$\textcircled{9} \quad n_1 = 12 \quad \bar{x} = 48.7 \quad n_2 = 7 \quad \bar{y} = 38.1$$

$$\sigma(x) = 7.65 \quad \sigma(y) = 4.2$$

$$(i) \quad \text{Total combined mean} = \bar{t} = \frac{\sum x + \sum y}{19}$$

$$\sum x = n_1 \bar{x} = (12)(48.7) = 584.4$$

$$\sum y = n_2 \bar{y} = (7)(38.1) = 266.7$$

$$\therefore \quad \bar{t} = \frac{584.4 + 266.7}{19}$$

$$\boxed{\bar{t} = 44.8}$$

$$(ii) \quad \sigma(x) = \sqrt{\frac{\sum x^2}{n_1} - \bar{x}^2}$$

$$\begin{aligned} \sum x^2 &= (\sigma^2(x) + \bar{x}^2) n_1 \\ &= (7.65^2 + 48.7^2)(12) \end{aligned}$$

$$\sum x^2 = 29,162.55$$

$$\begin{aligned}\sum y^2 &= (\sigma^2(x) + \bar{y}^2) n_2 \\ &= (4.2^2 + 38.1^2)(7)\end{aligned}$$

$$\sum y^2 = 10,284.75$$

$$\begin{aligned}\sigma^2(t) &= \frac{\sum x^2 + \sum y^2}{19} - \bar{t}^2 \\ &= \frac{29,162.55 + 10,284.75}{19} - 44.8^2\end{aligned}$$

$$\therefore \boxed{\sigma(t) = 8.37}$$

$$\textcircled{10} \quad n_x = 12 \quad \bar{x} = 15.5 \quad \sigma(x) = 1.2$$

$$n_y = 20 \quad \sum y = 910 \quad \sum y^2 = 42,850$$

$$(i) \quad \text{combined mean} = \bar{t} = \frac{\sum x + \sum y}{n_x + n_y}$$

$$\sum x = n_x \bar{x} = 12 \times 15.5 = 186$$

$$\therefore \bar{t} = \frac{186 + 910}{32} = 34.25$$

$$\boxed{\bar{t} = 34.25}$$

(ii) t is group of all 32 members.

$$\sigma^2(t) = \frac{\sum x^2 + \sum y^2}{32} - \bar{t}^2$$

$$\sigma^2(x) = \frac{\sum x^2}{n_x} - \bar{x}^2$$

$$\therefore \sum x^2 = (\sigma^2(x) + \bar{x}^2) n_x$$

$$= ((1.2)^2 + (15.5)^2)(12)$$

$$\therefore \sum x^2 = 2900.28$$

$$\therefore \sigma^2(t) = \frac{2900.28 + 42,850}{32} - (34.25)^2$$

$$\therefore \boxed{\sigma(t) = 16.02}$$

⑪ $n=20 \quad \bar{x}=60 \quad \sigma(x)=4$

(i)

$$\begin{aligned} \sum x &= n \bar{x} \\ &= 20 \times 60 \end{aligned}$$

$$\boxed{\sum x = 1200}$$

$$\sigma^2(x) = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\begin{aligned} \sum x^2 &= (\sigma^2(x) + \bar{x}^2) n \\ &= (4^2 + 60^2)(20) \end{aligned}$$

$$\therefore \boxed{\sum x^2 = 72,320}$$

$$(ii) \quad n_1 = 10 \quad \sum x_n = 550 \quad \sum x_n^2 = 40500$$

$$\begin{aligned} \text{COMBINED MEAN} &= \bar{t} = \frac{\sum x + \sum x_n}{30} \\ &= \frac{550 + 1200}{30} \end{aligned}$$

$$\therefore \boxed{\bar{t} = 58.33}$$

$$\begin{aligned} \sigma^2(t) &= \frac{\sum x^2 + \sum x_n^2}{30} - \bar{t}^2 \\ &= \frac{72,320 + 40,500}{30} - (58.33)^2 \end{aligned}$$

$$\therefore \boxed{\sigma(t) = 18.92}$$

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(i)

$$\text{Combined Mean} = \bar{X}_T = \frac{\sum x_t + \sum x_g}{27}$$

$$\sum x_t = n_t \bar{x}_t = 9 \times 7.1 = 63.9$$

$$\sum x_g = n_g \bar{x}_g = 18 \times 5.2 = 93.6$$

$$\therefore \bar{X}_T = \frac{63.9 + 93.6}{27} = 5.83$$

$$\boxed{\bar{X}_T = 5.83}$$

(ii)

$$\sigma^2(x_t) = \frac{\sum x_t^2}{n_t} - \bar{x}_t^2$$

$$\begin{aligned} \therefore \sum x_t^2 &= (\sigma^2(x_t) + \bar{x}_t^2) n_t \\ &= (1.45^2 + 7.1^2)(9) \end{aligned}$$

$$\therefore \sum x_t^2 = 472.61$$

$$\begin{aligned} \sum x_g^2 &= (\sigma^2(x_g) + \bar{x}_g^2) n_g \\ &= (0.96^2 + 5.2^2) 18 \end{aligned}$$

$$\therefore \sum x_g^2 = 503.31$$

$$\begin{aligned}\therefore \sigma^2(X_T) &= \frac{\sum x_t^2 + \sum x_g^2}{27} - \bar{X}_T^2 \\ &= \frac{472.61 + 503.31}{27} - 5.83^2\end{aligned}$$

$$\therefore \boxed{\sigma(X_T) = 1.46}$$

$$(13) \quad (i) \quad \bar{X}_T = \frac{\sum x_F + \sum x_L}{51}$$

$$\sum x_F = 30 \bar{x}_F = 30 \times 1500 = 45,000$$

$$\sum x_L = 21 \bar{x}_L = 21 \times 2400 = 50,400$$

$$\therefore \bar{X}_T = \frac{45,000 + 50,400}{51} = 1870.59$$

$$\boxed{\bar{X}_T = 1870.59}$$

$$\begin{aligned}
 \text{(ii)} \quad \sum x_F^2 &= (\sigma^2(x_F) + \bar{x}_F^2) \times 30 \\
 &= ((230)^2 + (1500)^2) \times 30 \\
 &= 6.9087 \times 10^7
 \end{aligned}$$

$$\begin{aligned}
 \sum x_L^2 &= (\sigma^2(x_L) + \bar{x}_L^2) \times 21 \\
 &= ((160)^2 + (2400)^2) \times 21 \\
 &= 1.2149 \times 10^8
 \end{aligned}$$

$$\begin{aligned}
 \therefore \sigma^2(x_T) &= \frac{\sum x_F^2 + \sum x_L^2}{51} - \bar{X}_T^2 \\
 &= \frac{6.9087 \times 10^7 + 1.2149 \times 10^8}{51} - (1870.59)^2
 \end{aligned}$$

$$\therefore \boxed{\sigma(x_T) = 487.7}$$

(14)

$$\begin{aligned}(i) \quad \sum (t-120) &= (95-120) + (126-120) + (117-120) \\ &\quad + (135-120) + (120-120) + (125-120) \\ &\quad + (114-120) + (119-120) + (136-120) \\ &= (-25) + (6) + (-3) + (15) \\ &\quad + (0) + (5) + (-6) + (-1) \\ &\quad + (16)\end{aligned}$$

$$\therefore \boxed{\sum (t-120) = 7}$$

$$\begin{aligned}\sum (t-120)^2 &= (-25)^2 + (6)^2 + (-3)^2 + \\ &\quad (15)^2 + (5)^2 + (-6)^2 + \\ &\quad (-1)^2 + (16)^2\end{aligned}$$

$$\boxed{\sum (t-120)^2 = 1213}$$

$$(ii) \quad \sigma^2(t-120) = \sigma^2(t)$$

$$\frac{\sum (t-120)^2}{9} - \bar{t}^2 = \sigma^2(t)$$

$$\therefore \sigma^2(t) = \frac{1213}{9} - \left(\frac{7}{9}\right)^2$$

$$\boxed{\sigma^2(t) = 134.2}$$

(15) $n = 11$ $\sum x = 1923$
 $\sum x^2 = 337,221$

$$n_T = 14 \quad \sum x_T = \sum x + 166 + 172 + 182$$

$$= 2443$$

$$\sum x_T^2 = \sum x^2 + 166^2 + 172^2 + 182^2$$

$$= 427,485$$

$$\sigma^2(x_T) = \frac{\sum x_T^2}{14} - \bar{x}_T^2$$

$$= \frac{427,485}{14} - \left(\frac{2443}{14}\right)^2$$

$$\therefore \boxed{\sigma(x_T) = 9.19}$$